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Five–branes and supersymmetry breaking in M–theory

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Abstract: Supersymmetry breaking via gaugino condensation is studied in vacua of heterotic M–theory with five–branes. We show that supersymmetry is still broken by a global mechanism and that the non–perturbative superpotential takes the standard form. When expressed in terms of low energy fields, a modification arises due to a threshold correction in the gauge kinetic function that depends on five–brane moduli. We also determine the form of the low energy matter field Kähler potential. These results are used to discuss the soft supersymmetry breaking parameters, in particular the question of universality.

Keywords: M-Theory, p-branes, Supersymmetry Breaking, Supersymmetric Effective Theories.

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Contents

1. Introduction

Starting with the seminal work by Hořava and Witten [1, 2, 3, 4], considerable activity [5]–[47] has been devoted to exploring various aspects of M–theory on the orbifold S^1/Z_2 which represents the strong coupling limit of $E_8 \times E_8$ heterotic string theory. Most of the work dealing with compactifications of this theory based on Calabi–Yau three–folds has used the standard–embedding of the spin connection into the gauge group as a starting point. Although this leads to particularly simple solutions of the weakly coupled heterotic string, this is not the case for its strongly coupled counterpart. In fact, there does not seem to be any way, for Calabi–Yau type compactifications, to set the antisymmetric tensor field of 11–dimensional supergravity to zero. Hence, the standard–embedding does not appear to be particularly special in the strong coupling limit and one is led to consider the general case of non–standard embeddings. Aspects of such non–standard embeddings, in vacua of heterotic M–theory not involving five–branes have been discussed in [32, 39, 40, 41].

It turns out that such non–standard embedding vacua are, in fact, a small subclass of a much larger class of vacua that, in addition, allows for the presence of five–branes. Including five–branes is quite natural from the M–theory viewpoint once general non–standard embeddings are allowed and is essential for a "complete" discussion of such vacua. These non–perturbative vacua, based on Calabi–Yau three–folds, general E_8 gauge bundles and a number of five–branes, have been constructed in two recent papers [41, 46]. The five–branes are oriented such that they stretch across the $3+1$–dimensional uncompactified space and wrap on two–cycles within the Calabi–Yau manifold.

These non–perturbative vacua have a number of new and perhaps surprising properties. Most notably, it has been explicitly shown [46] that three–generation models with attractive low–energy GUT groups exist for most Calabi–Yau spaces in a certain specific class. This is facilitated by the presence of the five–branes which introduce considerably more freedom in the anomaly cancellation condition that consistent vacua need to satisfy. These results suggest that three–generation models exist for most Calabi–Yau spaces once five–branes are allowed in the vacuum. Another interesting feature is the appearance of new sectors in the low energy theory that originate from the degrees of freedom on the five–brane worldvolumes. In particular, these new sectors carry Abelian or non–Abelian gauge theories that enlarge the total low energy gauge group. At low energy, all these sectors are coupled to one another and to the two conventional sectors that arise from $E_8 \times E_8$ only gravitationally. Another interesting feature is that the two E_8 sectors are modified due to the presence of the five–branes. For example, the gauge kinetic functions depend on the moduli which specify the positions of the five–branes in the orbifold direction. The appearance of these moduli is a non–perturbative effect that cannot be seen in the weakly coupled limit of the heterotic string. These properties make vacua of heterotic M–theory with five–branes a rich and interesting arena for particle physics.

In this paper, we would like to investigate some of the physical properties of these new non–perturbative vacua. Specifically, we will analyze in detail how supersymmetry breaking via gaugino condensation [54, 55] in the hidden sector works. For standard embedding vacua, gaugino condensation in heterotic M–theory was first analyzed by Hořava [3] and subsequently by [17, 22]. Supersymmetry breaking in five dimensions was also studied in [48, 33, 45]. Furthermore, we will calculate the matter field part of the $D = 4$ low energy effective action in its generic form. These results will then be used to discuss soft supersymmetry breaking terms and, in particular, the question of universality. Recently, supersymmetry breaking and universality of the soft breaking parameters in supergravity theories that originate from vacua with branes has also been discussed in ref. [44].

It is important to be clear about the interpretation of the vacua from the low energy viewpoint. In this paper, we will use a "conservative" interpretation in that we assume the observable sector to arise from one of the E_8 sectors. Consequently, the five–brane sectors, as well as the other E_8 sector, are hidden. In addition, we assume that supersymmetry breaking via gaugino condensation occurs in the hidden E_8 sector. These assumptions could be modified in two ways. First, it is conceivable that, under certain conditions, one of the five–brane sectors could also be used as the observable sector. Although models along those lines have yet to be constructed in heterotic M–theory, there are recent type I examples [49, 50, 51] where semi–realistic low–energy theories arise from three–brane worldvolume theories. In any case, the relation of our constructions to brane–box models [52, 53] suggests that chiral theories from the five–brane sectors might exist. As a second modification, gaugino condensation could also occur in those hidden five–brane sectors that carry non–Abelian gauge groups. Although these modifications are worth investigating, in this paper we stick to the simple interpretation explained above. In the end, we briefly comment on gaugino condensation in the five–brane sectors.

An important question we would like to address is how the presence of the five–branes affects supersymmetry breaking via gaugino condensation. For the standard–embedding case, Hořava has shown that supersymmetry is broken by a mechanism that is global in the orbifold direction. Despite this new feature, the non–perturbative low energy potential induced by this condensate turned out to be the same as in the weakly coupled case [5, 22]. The five–branes present in our non–perturbative vacua are stacked up along the orbifold direction; that is, precisely the direction in which the global breaking mechanism occurs. It is, therefore, quite conceivable that the five–branes can significantly alter the picture of supersymmetry breaking via gaugino condensation.

To analyze this in detail, we will first review the form of the vacua with five–branes, both from a topological and an analytic viewpoint. In particular, we present the explicit form of those vacua as an expansion, to linear order, in the strong coupling expansion parameter $\epsilon = (\kappa/4\pi)^{2/3} 2\pi^2 \rho / v^{2/3}$. Here κ is the eleven–dimensional Newton constant, v is the Calabi–Yau volume and ρ is the orbifold radius. Subsequently, we incorporate gaugino condensation and analyze how supersymmetry is broken on the level of the vacuum solution. In section 3, we compute the $D = 4$ superpotential from gaugino condensation in terms of the condensate. While this was done in the 11–dimensional theory, in section 4, we present an alternative derivation in the five–dimensional effective theory of heterotic M–theory. To express the superpotential in terms of low energy fields, we need to compute the matter field part of the effective four–dimensional theory which is done in section 5. Finally, in section 6, we use these results to discuss soft supersymmetry breaking terms.

Our results can be summarized as follows. On the level of the pure vacuum solution, supersymmetry breaking via gaugino condensation in the presence of five–branes still occurs via Hořava's global mechanism. Furthermore, the condensate superpotential as expressed in terms of the condensate is unchanged with respect to the ordinary case. Roughly speaking, these results can be attributed to the fact that the gaugino condensate is associated with a different sector of the Calabi–Yau space than the curves on which the five–branes are wrapped. Once the non–perturbative condensate potential is expressed in terms of the moduli fields, a new feature related to the presence of five–branes occurs. It turns out that the gauge kinetic functions in the E_8 sectors have threshold corrections of order ϵ which not only depend on the T moduli, but also on the moduli that specify the positions of the five–branes in the orbifold direction [41]. Hence, the potential not only depends on the dilaton S and the T–moduli T^i, but also on the five–brane position moduli Z^n. A similar correction of order ϵ to the matter field Kähler metric depending on

S and Z^n is found as well. Based on these results, we propose a parameterization of supersymmetry breaking in terms of the auxiliary fields associated to S, T^i and the five–brane moduli Z^n. Using this parameterization, we analyze the question of universality of soft parameters. Universality can be achieved at zeroth order in ϵ by assuming supersymmetry breaking in the S or Z^n direction. As usual, breaking in the T^i directions is not generation–independent, even at zeroth order. From an 11–dimensional point of view this happens because both the T–moduli and the matter fields arise as zero modes of the Calabi–Yau space. When the order ϵ corrections to the matter field Kähler metric are taken into account no specific supersymmetry breaking pattern leads to universality. Generally, not even a pure S–type breaking scenario [60] will work. The 11–dimensional reason for this is an orbifold dependence of the background on which the reduction to four dimensions is performed. This dependence arises from the sources provided by the fields on the orbifold planes and the five–branes and, as such, is a general phenomenon for vacua containing branes. We observe that this difficulty can be overcome by choosing a specific type of Calabi–Yau space with $h^{1,1} = 1$. In this case, universality to order ϵ is guaranteed independently of the breaking pattern. Soft breaking parameters are then computed for this particular case. These parameters can be of the order $m_{3/2}$ or $\epsilon m_{3/2}$ with a characteristic signature that is directly correlated to the pattern of supersymmetry breaking.

2. The eleven–dimensional background

In this section, we would like to review the vacua of heterotic M–theory with non–standard embedding and five–branes constructed in ref. [41, 46]. In addition, we would like to incorporate gaugino condensates on the orbifold planes and show, on the level of the vacuum solution, how supersymmetry is broken by those condensates. Most importantly, we would like to understand the role of the five–branes in this context. None of the ingredients needed for this is essentially new. Nevertheless we find it useful to present a coherent argument and to set the stage for what is going to follow.

Let us first explain the basic structure of the non–perturbative vacua and their topological properties following ref. [46]. To lowest order, we consider a space–time

$$M_{11} = M_4 \times S^1/\mathbb{Z}_2\,, \times X \tag{2.1}$$

where X is a Calabi–Yau three–fold and M_4 is four–dimensional Minkowski space. Furthermore, one needs to specify two semi–stable gauge bundles V_i, $i = 1, 2$ on X with gauge groups $G_i \subset E_8$. As discussed, we also allow for the presence of N five–branes in the vacuum. These five–branes should stretch across M_4 (to preserve 3+1–

dimensional Poincaré invariance) and wrap on holomorphic curves $C^{(n)} \subset X$ where $n = 1, \ldots, N$ (to preserve $\mathcal{N} = 1$ supersymmetry in four dimensions). Note that, given this orientation, the five–branes are located at specific points in the orbifold direction which we denote by $x^{11} = x_1, \ldots, x_N$, where $0 \leq x_1 \leq \cdots \leq x_N \leq \pi\rho$. As we will discuss later on, inclusion of five–branes is important as it allows for the construction of physically interesting vacua that would otherwise be forbidden. Heuristically, this can be understood from the cohomology constraint

$$c_2(V_1) + c_2(V_2) + [W] = c_2(TX) \tag{2.2}$$

that has to be imposed on such vacua in order to be anomaly free. Here $[W]$ is the cohomology class associated with the five–branes and $c_2(V_i)$ and $c_2(TX)$ are the second Chern classes of the gauge bundles V_i and the tangent bundle TX respectively. In fact, for a given Calabi–Yau space and given vector bundles one can always choose a class $[W]$ such that eq. (2.2) is satisfied. As was shown in ref. [46], there is an additional physical constraint on $[W]$, namely that it constitutes an effective class. Essentially, this means that $[W]$ corresponds to holomorphic curves in X; that is, the associated five–brane must be of the form $W = \sum_{n=1}^{N} C^{(n)}$ where the $C^{(n)}$ are linear combinations of holomorphic curves with non–negative integer coefficients. This has to be contrasted to the situation without five–branes where simply $[W] = 0$.

In ref. [46], these statements have been made much more precise by constructing $\mathrm{SU}(n)$ gauge bundles over elliptically fibered Calabi–Yau spaces. This allows one to explicitly compute the Chern–classes $c_2(TX)$ and $c_2(V_i)$ and hence, from eq. (2.2), the class $[W]$ of the five–brane curve. It was shown that the effectiveness constraint can be satisfied and further physical requirements can be imposed. In particular, a three–generation constraint can be accommodated without problems along with, for example, an SU(5) gauge bundle that leads to a low energy SU(5) grand unification group. It was also demonstrated, for a specific example, how to determine the moduli space of curves W associated with the class $[W]$. This moduli space turns out to be a complicated multi–branched object that carries information about the additional five–branes sectors. In particular, sticking to a specific branch, it allows one to read off the number of five–branes, N, and the classes $[C^{(n)}]$ of their associated curves. With this information one can define the instanton numbers $\beta_i^{(0)}$, $\beta_i^{(N+1)}$ and the five–brane charges $\beta_i^{(n)}$, $n = 1, \ldots, N$ by

$$\begin{aligned} c_2(V_1) - \frac{1}{2}c_2(TX) &= \beta_i^{(0)} C_2^i \\ c_2(V_2) - \frac{1}{2}c_2(TX) &= \beta_i^{(N+1)} C_2^i \\ \left[C^{(n)}\right] &= \beta_i^{(n)} C_2^i \,. \end{aligned} \tag{2.3}$$

Here $\{C_2^i\}$ is a basis of the second homology group $H_2(X,\mathbb{Z})$ (which we identify with $H^4(X,\mathbb{Z})$ via Poincaré duality). The cohomology constraint (2.2) then takes the form

$$\sum_{n=0}^{N+1} \beta_i^{(n)} = 0 \tag{2.4}$$

for all $i = 1, \ldots, h^{1,1}$. The numbers $\beta_i^{(n)}$ constitute the essential topological information that appears in the explicit form of the vacua.

Let us now review this explicit form following ref. [41]. Generally, an 11–dimensional M–theory vacuum is specified by the metric g_{IJ} and the three–form C_{IJK} with field strength $G_{IJKL} = 24\,\partial_{[I}C_{JKL]}$. For our discussion of supersymmetry breaking, we will also need the Killing spinor η corresponding to four preserved supercharges. We will use 11–dimensional indices $I, J, K, \ldots = 0, \ldots, 9, 11$, four–dimensional indices $\mu, \nu, \rho, \ldots = 0, \ldots 3$ and Calabi–Yau indices $A, B, C, \ldots = 4, \ldots, 9$. Holomorphic and anti–holomorphic indices on the Calabi–Yau space are denoted by $a, b, c, \ldots$ and $\bar{a}, \bar{b}, \bar{c}, \ldots$. The form of the vacuum has been computed using an expansion, to linear order, in the parameter

$$\epsilon = \left(\frac{\kappa}{4\pi}\right)^{2/3} \frac{2\pi^2 \rho}{v^{2/3}}\,. \tag{2.5}$$

Here κ is the 11–dimensional Newton constant, v is the Calabi–Yau volume and ρ is the orbifold radius. To this order the solution takes the form [4]

$$\begin{aligned}
ds^2 &= (1+b)\eta_{\mu\nu}dx^\mu dx^\nu + (g_{AB}^{(\mathrm{CY})} + h_{AB})dx^A dx^B + (1+\gamma)(dx^{11})^2 \\
G_{ABCD} &= G_{ABCD}^{(1)} \\
G_{ABC11} &= G_{ABC11}^{(1)} \\
\eta &= (1+\psi)\eta^{(\mathrm{CY})}
\end{aligned} \tag{2.6}$$

and all other components of G vanishing. Here $g_{AB}^{(\mathrm{CY})}$ and $\eta^{(\mathrm{CY})}$ are the Ricci–flat metric and the covariantly constant spinor on the Calabi–Yau space. The quantities b, h_{AB}, γ, $G^{(1)}$ and ψ represent the corrections linear in ϵ. Following ref. [20] they can be expressed in terms of a single $(1,1)$–form $\mathcal{B}_{a\bar{b}}$ on X as follows

$$\begin{aligned}
h_{a\bar{b}} &= \sqrt{2}i\left(\mathcal{B}_{a\bar{b}} - \frac{1}{3}\omega_{a\bar{b}}\mathcal{B}\right) \\
b &= \frac{\sqrt{2}}{6}\mathcal{B}\,, \qquad \gamma = -\frac{\sqrt{2}}{3}\mathcal{B}\,, \qquad \psi = -\frac{\sqrt{2}}{24}\mathcal{B} \\
G_{ABCD}^{(1)} &= \frac{1}{2}\epsilon_{ABCDEF}\partial_{11}\mathcal{B}^{EF} \\
G_{ABC11}^{(1)} &= \frac{1}{2}\epsilon_{ABCDEF}\partial^D\mathcal{B}^{EF}\,.
\end{aligned} \tag{2.7}$$

This $(1,1)$–form can be expanded in terms of harmonics of the Calabi–Yau laplacean. For the purpose of computing low energy effective actions, it is sufficient to keep the massless terms in this expansion; that is, the terms proportional to the harmonic $(1,1)$–forms of the Calabi–Yau space. Let us choose a basis $\{\omega_{ia\bar{b}}\}$ of those forms dual to the cycles C_2^i introduced above. We then write

$$\mathcal{B}_{a\bar{b}} = \sum_i b_i \omega^i_{a\bar{b}} + (\text{massive terms})\,, \tag{2.8}$$

where the indices are lowered and raised with the metric

$$G_{ij} = \frac{1}{2v}\int_X \omega_i \wedge (*\omega_j) \tag{2.9}$$

on the Kähler moduli space. The expansion coefficients b_i are given by [41]

$$b_i = \frac{\epsilon}{\sqrt{2}}\left[\sum_{m=0}^{n} \beta_i^{(m)}(|z| - z^m) - \frac{1}{2}\sum_{m=0}^{N+1}(1 - z^m)^2\beta_i^{(m)}\right] \tag{2.10}$$

in the interval

$$z^n \leq |z| \leq z^{n+1}\,, \tag{2.11}$$

for fixed n, where $n = 0, \ldots, N$. Here $z = x^{11}/\pi\rho \in [-1,1]$ is the normalized orbifold coordinate and $z^n = x_n/\pi\rho \in [0,1]$ where $n = 1, \ldots, N$ are the five–brane positions in this coordinate frame. We have also defined $z_0 = 0$ and $z_{N+1} = 1$. To summarize, eqs. (2.6)–(2.10) determine the background to linear order in ϵ. As promised, the only input needed are the topological numbers $\beta_i^{(n)}$ defined in eq. (2.3). As we have discussed above, they can computed by methods of algebraic geometry, at least for certain classes of Calabi–Yau spaces.

Let us now analyze the properties of these vacua in the presence of gaugino condensates. We recall that, following Hořava [3], the proper way to avoid singularities in the presence of condensates is to redefine the antisymmetric tensor field according to

$$\begin{aligned}
\tilde{G}_{\bar{I}\bar{J}\bar{K}\bar{L}} &= G_{\bar{I}\bar{J}\bar{K}\bar{L}} \\
\tilde{G}_{\bar{I}\bar{J}\bar{K}11} &= G_{\bar{I}\bar{J}\bar{K}11} - \frac{\sqrt{2}}{16\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\left(\delta(x^{11})\omega^{(\chi,1)} + \delta(x^{11} - \pi\rho)\omega^{(\chi,2)}\right)_{\bar{I}\bar{J}\bar{K}}\,,
\end{aligned} \tag{2.12}$$

with the gaugino bilinears

$$\omega^{(\chi,i)}_{\bar{I}\bar{J}\bar{K}} = \mathrm{tr}\,\bar{\chi}^{(i)}\Gamma_{\bar{I}\bar{J}\bar{K}}\chi^{(i)}\,. \tag{2.13}$$

In terms of the new field $\tilde{G}$ the relevant part of the 11–dimensional action takes the simple form

$$S^{(\chi)} = -\frac{1}{48\kappa^2}\int_{M_{11}} \sqrt{-g}\,\tilde{G}_{IJKL}\tilde{G}^{IJKL}, \tag{2.14}$$

which reflects the familiar fact that terms containing gaugino bilinears, together with the original G^2–term, group into a perfect square. In addition, we need the Killing spinor equations in terms of $\tilde{G}$

$$\delta\Psi_{\bar{I}} = D_{\bar{I}}\eta + \frac{\sqrt{2}}{288}\tilde{G}_{JKLM}\left(\Gamma_{\bar{I}}{}^{JKLM} - 8\delta_{\bar{I}}^{J}\Gamma^{KLM}\right)\eta = 0 \tag{2.15}$$

$$\begin{aligned}\delta\Psi_{11} = {} & D_{11}\eta + \frac{\sqrt{2}}{288}\tilde{G}_{JKLM}\left(\Gamma_{11}{}^{JKLM} - 8\delta_{11}^{J}\Gamma^{KLM}\right)\eta + \\ & + \frac{1}{192\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\left[\delta(x^{11})\omega^{(\chi,1)} + \delta(x^{11}-\pi\rho)\omega^{(\chi,2)}\right]_{\bar{I}\bar{J}\bar{K}}\Gamma^{\bar{I}\bar{J}\bar{K}}\eta = 0\end{aligned} \tag{2.16}$$

and the equations of motion and the Bianchi identity for $\tilde{G}$

$$D_I\tilde{G}^{IJKL} = 0 \tag{2.17}$$

$$\begin{aligned}(d\tilde{G})_{11\bar{I}\bar{J}\bar{K}\bar{L}} = {} & 4\sqrt{2}\pi\left(\frac{\kappa}{4\pi}\right)^{2/3}\Bigg[J^{(0)}\delta(x^{11}) + J^{(N+1)}\delta(x^{11}-\pi\rho) + \\ & + \frac{1}{2}\sum_{n=1}^{N} J^{(n)}(\delta(x^{11}-x_n) + \delta(x^{11}+x_n))\Bigg]_{\bar{I}\bar{J}\bar{K}\bar{L}} - \\ & - \frac{\sqrt{2}}{16\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\left[J^{(\chi,1)}\delta(x^{11}) + J^{(\chi,2)}\delta(x^{11}-\pi\rho)\right]_{\bar{I}\bar{J}\bar{K}\bar{L}}\,.\end{aligned} \tag{2.18}$$

The sources $J^{(0)}$ and $J^{(N+1)}$ on the orbifold planes are as usual given by

$$\begin{aligned}J^{(0)} &= -\frac{1}{16\pi^2}\left(\mathrm{tr}F^{(1)}\wedge F^{(1)} - \frac{1}{2}\mathrm{tr}R\wedge R\right)\Bigg|_{x^{11}=0}\,, \\ J^{(N+1)} &= -\frac{1}{16\pi^2}\left(\mathrm{tr}F^{(2)}\wedge F^{(2)} - \frac{1}{2}\mathrm{tr}R\wedge R\right)\Bigg|_{x^{11}=\pi\rho}\,.\end{aligned} \tag{2.19}$$

The sources $J^{(n)}$, $n = 1,\ldots,N$ arise due to the presence of the five–branes. Finally, the sources $J^{(\chi,i)}$ originate from the gaugino condensates and are defined as

$$J^{(\chi,i)} = d\omega^{(\chi,i)}\,. \tag{2.20}$$

For generality, we have considered condensates on both orbifold planes. Later on we will specialize to the "physical" case of one condensate only. Specializing to the field configuration of the above vacua all sources $J^{(n)}$, where $n = 0,\ldots,N+1$, constitute $(2,2)$–forms on the Calabi–Yau space. Then the charges $\beta_i^{(n)}$ that we have defined earlier can be written as

$$\beta_i^{(n)} = \int_{C_{4i}} J^{(n)}\,. \tag{2.21}$$

where $\{C_{4i}\}$ is a basis of $H_4(X)$ dual to the basis $\{C_2^i\}$ of $H_2(X)$ introduced above.

Of course, the non–standard embedding vacua with five–branes, eq. (2.6)–(2.10), that we have constructed constitute a solution of the eqs. (2.15)–(2.18) in the absence of condensates, as they should. As usual, we will consider condensates which, as part of the vacuum, are covariantly constant and closed. Let us now discuss our vacua in the presence of such condensates. The only change in the equations (with the replacement of G by $\tilde{G}$ understood) is then the appearance of the condensate in the Killing spinor equation (2.16). Hence, if we set $\tilde{G}$ equal to the field $G^{(1)}$ of our solutions and use the corresponding Killing spinor η and metric g_{IJ}, we can satisfy all equations except $\delta\Psi_{11} = 0$. Following ref. [3], one can try and modify the spinor η to $\eta + \delta\eta$ so as to also satisfy this final equation. One finds from eq. (2.16)

$$\partial_{11}(\delta\eta) = -\frac{1}{192\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\left[\omega^{(\chi,1)}\delta(x^{11}) + \omega^{(\chi,2)}\delta(x^{11} - \pi\rho)\right]_{\bar{I}\bar{J}\bar{K}}\Gamma^{\bar{I}\bar{J}\bar{K}}\eta_0 \,. \qquad (2.22)$$

Locally, this equation is solved by $\delta\eta$ proportional to $\omega^{(\chi,i)}_{\bar{I}\bar{J}\bar{K}}\Gamma^{\bar{I}\bar{J}\bar{K}}\eta^{(\mathrm{CY})}\epsilon(x^{11})$, where $\epsilon(x^{11})$ is the step functions which is $+1$ for positive x^{11} and -1 otherwise. Such a modification also leaves the 10–dimensional part of the Killing spinor equation, $\delta\Psi_{\bar{I}} = 0$, intact since the condensates are covariantly constant. Matching both δ–functions in eq. (2.22), however, is impossible unless

$$(\omega^{(\chi,1)} + \omega^{(\chi,2)})_{\bar{I}\bar{J}\bar{K}}\Gamma^{\bar{I}\bar{J}\bar{K}}\eta_0 = 0\,. \qquad (2.23)$$

Generally, this condition is violated and this is particularly so in the "physical" situation of a condensate on one orbifold plane only. Clearly, the obstruction of solving eq. (2.23) is global in nature and is, of course, just the "global" supersymmetry breaking mechanism discovered by Hořava [3]. What we have shown here is that this global mechanism continues to work even for the more general vacua with non–standard embedding and five–branes. This is not surprising since the essential part of the argument does not depend on the specific form of the linearized solution considered, but merely on the fact that the condensates are covariantly constant and closed. Let us summarize the essential steps of the argument. We have seen that the five–brane sources $J^{(n)}$ enter the problem only via the Bianchi identity (2.18). The condensate sources $J^{(\chi,i)}$ in this Bianchi identity vanish since the condensate is closed. Therefore, the redefined antisymmetric tensor field $\tilde{G}$ is unchanged from the previous G. As a consequence, the only obstruction to satisfying the Killing spinor equations comes from the explicit condensate terms in eq. (2.16). Hence, the equation (2.22) for the correction to the Killing spinor does not contain the antisymmetric tensor field $\tilde{G}$, the only object that carries information about the five–branes. The nature of the breaking is, therefore, independent on the presence of five–branes. Although not involving anything essentially new, we found it worth presenting the above argument in some detail since it contradicts the "naive" expectation. Intuitively, one might expect that a supersymmetry breaking mechanism global in the orbifold is modified by five–branes stacked along this direction. What we have seen is that this does not happen, at least for the pure background case that we have discussed so far.

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3. The gaugino condensate potential

In the previous section, we have shown that even for non–standard embedding and in the presence of five–branes supersymmetry breaking due to gaugino condensates on the orbifold planes continues to work via Hořava's global mechanism. Now we would like to proceed further and compute the low energy superpotential associated with this breaking. Again, an important question to be clarified is how the five–branes enter the four–dimensional low energy theory. In particular, since the five–branes are stacked "between" what will become the hidden and the observable sector at low energy, one might expect a significant change of the non–perturbative potential.

To address this problem, let us first recall how the gaugino condensate superpotential arises in heterotic M–theory [22]. Although the condensate sources $J^{(\chi,i)}$ vanish for the background solution, this is no longer true once the moduli in this solution are promoted to four–dimensional fields. More specifically, we use the standard expression [55]

$$\omega^{(\chi,i)}_{abc} = \mathrm{tr}\,\bar{\chi}^{(i)}\Gamma_{abc}\chi^{(i)} = \Lambda^{(i)}\Omega_{abc} \tag{3.1}$$

for the condensate, where Ω_{abc} is the covariantly constant $(3,0)$–form of the Calabi–Yau space. In this expression, the condensation scales $\Lambda^{(i)}$ are functions of the low–energy moduli fields. Hence, we have the non–vanishing components

$$J^{(\chi,i)}_{\mu abc} = \partial_\mu\Lambda^{(i)}\Omega_{abc} \tag{3.2}$$

of the gaugino sources. In the Bianchi identity (2.18), these sources appear with δ–functions and they have to be carefully integrated out when deriving the four–dimensional effective action. As was first observed in ref. [20, 24], this is a general phenomenon for theories with dynamical fields on boundaries. Not only have boundary sources induced by the background fields to be taken into account (as we did when we derived the above vacuum solutions) but, in addition, all sources that appear once the zero modes are promoted to fields in the low energy theory. Doing this is essential in order to arrive at the correct low energy theory and, in particular, to obtain all terms required by supersymmetry. Applied to the present situation, it means that the antisymmetric tensor field $\tilde{G}$ should have the form

$$\tilde{G} = G^{(1)} + G^{(\mathrm{mod})} + G^{(\chi)}\,. \tag{3.3}$$

Here $G^{(1)}$ is the background part explicitly given in eq. (2.7) and $G^{(\chi)}$ originates from integrating out the condensate sources (3.2) in the Bianchi identity (2.18). Explicitly one has

$$G^{(\chi)}_{abc11} = -\frac{\sqrt{2}}{32\pi^2\rho}\left(\frac{\kappa}{4\pi}\right)^{2/3}\left[\Lambda^{(1)}+\Lambda^{(2)}+\lambda\right]\Omega_{abc} \tag{3.4}$$

$$G^{(\chi)}_{\mu abc} = -\frac{\sqrt{2}}{32\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\partial_\mu\left[\Lambda^{(1)}-\frac{|x^{11}|}{\pi\rho}(\Lambda^{(1)}+\Lambda^{(2)})\right]\Omega_{abc}\,. \tag{3.5}$$

Using the flux quantization rule of heterotic M–theory derived in ref. [22], one can show that the constant λ in this expression must vanish. Finally, $G^{(\mathrm{mod})}$ accounts for all other sources involving derivatives of low energy fields as discussed above. It is clear that the gaugino condensate potential arises by inserting $\tilde{G}$ into the perfect square (2.14). Then, the standard result for the low energy potential, of course, arises from $G^{(\chi)}_{abc11}G^{(\chi)abc11}$. One could, however, expect corrections to this result from mixing terms between $G^{(\chi)}$ and other parts of (3.3), in particular those that originate from the presence of the five–branes or contain five–brane zero modes. To discuss this in more detail, let us refer to a part of $\tilde{G}$ with n holomorphic, m antiholomorphic and k 11 indices (where $k = 0, 1$) as a (n, m, k) component. In this terminology, the standard gaugino potential arises from $(3, 0, 1)$ and $(0, 3, 1)$ components. Let us first consider the background field $G^{(1)}$. Since the form of $G^{(1)}$ is intimately related to the presence of the five–branes, a mixing term $G^{(1)}G^{(\chi)}$ could lead to a correction of the potential induced by five–branes. However, eq. (2.7) shows that that $G^{(1)}$ consists of $(2, 2, 0)$, (2,1,1) and (1,2,1) components only and, hence, such a mixing term is impossible. A part of $G^{(\mathrm{mod})}$ is related to sources from low energy fields on the orbifold planes. Explicit expressions for the various components from such sources have been given in ref. [20]. There exists, in fact, one $(3, 0, 1)$ component proportional to $\Omega_{abc}W$, where W is the matter field superpotential. This leads to an expected cross term between the condensate potential and the matter field superpotential. In fact, such a term always has to be present and is in no way related to the presence of five–branes. There is another part of $G^{(\mathrm{mod})}$ due to sources from low energy fields that originate from the five–brane worldvolume. The orientation of the five–branes guarantees, however, that this part of $G^{(\mathrm{mod})}$ is of the type $(2, 2, 0)$ and, hence, cannot mix with $G^{(\chi)}$. In conclusion, we find that the gaugino condensate potential does not receive corrections due to non–standard embedding or the presence of five–branes. We simply have $W_{\mathrm{gaugino}} \sim \Lambda^{(1)} + \Lambda^{(2)}$.

Let us now specialize to the physical case $\Lambda^{(1)} = 0$ and $\Lambda^{(2)} \neq 0$. Following [55], we write

$$\Lambda^{(2)} \sim \frac{\alpha_{\mathrm{GUT}}}{\sqrt{v}} \exp\left[-\frac{6\pi}{b\alpha_{\mathrm{GUT}}} f^{(2)}\right], \tag{3.6}$$

where

$$\alpha_{\mathrm{GUT}} = \frac{(4\pi\kappa^2)^{2/3}}{2v}\,. \tag{3.7}$$

Here b and $f^{(2)}$ are the β–function coefficient and the gauge kinetic function for the gauge group on the second orbifold plane. Hence the superpotential for a single condensate is given by

$$W_{\mathrm{gaugino}} = h \exp\left[-\frac{6\pi}{b\alpha_{\mathrm{GUT}}} f^{(2)}\right], \tag{3.8}$$

where h is a constant of order $\kappa/\rho^{1/2}v$. For more than one condensate [56], we get a sum of terms of the above type with different coefficients b, as usual.

To summarize, let us briefly recapitulate why five–branes did not effect the potential. Once the zero modes are promoted to low energy fields, the Bianchi identity (2.18) contains, in addition to the five–brane sources, condensate sources as well as other sources related to the zero modes on the orbifold planes and on the five–brane worldvolumes. This leads to a complicated structure of $\tilde{G}$ as indicated by eq. (3.3). The conventional part of the gaugino condensate potential results from $\tilde{G}^2$. Hence, one could expect cross terms between the gaugino part and parts related to the presence of five–branes. As we have seen, those cross terms do not exist because the gaugino part and the five–brane part are associated to different index structures in the Calabi–Yau space. As a result, the potential is unmodified.

4. The five–dimensional viewpoint

The above line of reasoning can be equivalently presented purely in the five–dimensional effective action of heterotic M–theory. Logically, this does not, of course, provide any new information over what we have done in 11 dimensions. However, since the five–dimensional effective action is of interest in its own right, we find it useful to reformulate our argument from a five–dimensional viewpoint.

To do so, we need to recall some facts about the five–dimensional effective action of heterotic M–theory [30, 36, 41]. The bulk part of this theory represents a gauged $\mathcal{N}=1$ supergravity theory coupled to $h^{1,1}-1$ vector multiplets and $h^{2,1}+1$ hypermultiplets. Five–dimensional space–time has the structure $M_5 = S^1/Z_2 \times M_4$ where M_4 is a smooth four manifold.

This bulk supergravity is then coupled to two four–dimensional $\mathcal{N}=1$ theories on the orbifold fix points that originate from the two E_8 sectors of the 11–dimensional theory and, in addition, to N three–brane worldvolume theories with $\mathcal{N}=1$ supersymmetry that originate from the five–branes in the vacuum. For the purpose of this section, we can concentrate on the structure of the bulk theory. Some details about the spectrum on the four–dimensional theories will be given in the next section.

To be specific, we introduce the Calabi–Yau $(1,1)$ moduli a_5^i and associated $(1,1)$ vector fields $\mathcal{A}_\alpha^i$, where $i,j,k,\ldots = 1,\ldots,h^{1,1}$. We use indices $\alpha,\beta,\gamma = 0,\ldots,3,11$ for the five–dimensional space–time. In the bulk, the field strengths of these vector fields are given by

$$\mathcal{F}^i_{\alpha\beta} = \partial_\alpha \mathcal{A}^i_\beta - \partial_\beta \mathcal{A}^i_\alpha \,. \tag{4.1}$$

The volume modulus of the Calabi–Yau space can be expressed in terms of the $(1,1)$ moduli as

$$V_5 = \frac{d_{ijk} a_5^i a_5^j a_5^k}{6} \,, \tag{4.2}$$

with the triple intersection numbers d_{ijk}. The multiplet structure in five dimensions also leads us to define the shape moduli

$$b_5^i = V_5^{-1/3} a_5^i \,. \tag{4.3}$$

Since they satisfy the constraint

$$\mathcal{K}(b_5) \equiv d_{ijk} b_5^i b_5^j b_5^k = 6 \,, \tag{4.4}$$

they represent only $h^{1,1} - 1$ independent degrees of freedom. To make this more explicit, we introduce independent scalars ϕ^x, where $x = 1, \ldots, h^{1,1} - 1$ such that $b_5^i = b_5^i(\phi^x)$. With these definitions, the gravity multiplet has the field content

$$(g_{\alpha\beta}, \frac{2}{3} b_{5i} \mathcal{A}_\alpha^i, \psi_\alpha^A) \,. \tag{4.5}$$

Here $g_{\alpha\beta}$ is the $D = 5$ metric, $\frac{2}{3} b_{5i} \mathcal{A}_\alpha^i$ is the graviphoton and ψ_α^A is the gravitino. Fermions in five dimensions are described by symplectic spinors and, in this section, we use indices $A, B, C, \ldots = 1, 2$ for the SU(2) R symmetry. Since the gravity multiplet contains one vector field one remains with $h^{1,1} - 1$ vector multiplets. Their structure is given by

$$(\phi^x, b_{5i}^x \mathcal{A}_\alpha^i, \lambda^{xA}) \tag{4.6}$$

where b_{5i}^x projects onto the ϕ^x subspace and λ^{xA} are the gauginos. Note that so far we have not used the volume modulus V_5. It becomes part of the universal hypermultiplet

$$(V_5, \sigma, \xi, \bar{\xi}, \zeta^A) \,. \tag{4.7}$$

Here σ is a real scalar and ζ^A are the hypermultiplet fermions. The bulk field strength of the complex scalar field ξ is denoted by

$$X_\alpha = \partial_\alpha \xi \,. \tag{4.8}$$

This field strength will be of some importance in the following. We do not need to explicitly introduce the remaining $h^{2,1}$ hypermultiplets.

The gauging of the theory is with respect to the isometry $\sigma \to \sigma + \text{const}$ of the universal hypermultiplet coset space and the associated gauge boson is a certain linear combination $\alpha_i \mathcal{A}_\alpha^i$ of the vectors fields, where α_i are charges. The bulk action for those fields has been derived in ref. [30, 36] for the case of the standard embedding. It has been shown [41] that this action remains valid even in the presence of five–branes once the charges α_i are chosen appropriately. More specifically, for the interval $z_n \leq |z| \leq z_{n+1}$ between any two neighboring five–branes, where $n = 0, \ldots, N$, one should replace the charges α_i by

$$\alpha_i \to \alpha_i^{(n)} \equiv \frac{\sqrt{2}\epsilon}{\rho} \sum_{m=0}^{n} \beta_i^{(n)} \,, \tag{4.9}$$

with $\beta_i^{(n)}$ as introduced in eq. (2.3). Strictly speaking, for each interval one has, therefore, a different gauged supergravity theory characterized by the appropriate set of gauge charges.

Now, we need to set up the essential parts of the $D = 5$ theory incorporating a gaugino condensate. As in 11 dimensions, we start with a field redefinition to avoid singularities. Since the condensate originates from the $(3, 0)$ part of the Calabi–Yau space, as in eq. (3.1), we need to redefine the corresponding zero mode. This zero mode is precisely the complex scalar field ξ with field strength X_α in the universal hypermultiplet. In analogy with eq. (2.12), we define

$$\tilde{X}_\mu = X_\mu \tag{4.10}$$

$$\tilde{X}_{11} = X_{11} + \frac{\sqrt{2}}{32\pi}\frac{\kappa_5^2}{\alpha_{\rm GUT}}\left[\lambda^{(1)}\delta(x^{11}) + \lambda^{(2)}\delta(x^{11} - \pi\rho)\right], \tag{4.11}$$

where $\kappa_5^2 = \kappa^2/v$ is the five–dimensional Newton constant and $\alpha_{\rm GUT} = (4\pi\kappa^2)^{2/3}/2v$. Then the analog in five dimensions of the perfect square (2.14) reads

$$S_5^{(X)} = -\frac{1}{\kappa_5^2}\int_{M_5}\sqrt{-g}\,V^{-1}\tilde{X}_\alpha\tilde{X}^\alpha\,. \tag{4.12}$$

The Bianchi identity for X_α has been given in ref. [36]. Inserting the definition (4.11) we obtain

$$\left(d\tilde{X}\right)_{11\mu} = -\frac{\kappa_5^2}{16\sqrt{2}\pi\alpha_{\rm GUT}}\left[(4J_\mu^{(1)} + \partial_\mu\Lambda^{(1)})\delta(x^{11}) + (4J_\mu^{(2)} + \partial_\mu\Lambda^{(2)})\delta(x^{11} - \pi\rho)\right], \tag{4.13}$$

where $J_\mu^{(i)}$ is proportional to $\partial_\mu W^{(i)}$, the derivative of the matter field superpotential on the i–th orbifold plane. It is important to note that neither the Bianchi identity (4.13) nor the relevant part of the equation of motion for $\tilde{X}_\alpha$ (to be derived from (4.12)) have sources located on the three–branes that originate from the five–branes. The reason for this was already explained from the 11–dimensional perspective. Due to the orientation of the five–branes, they cannot contribute to the $(3, 0)$ part of the Calabi–Yau space. In the five–dimensional theory, this fact is manifest in the structure of the above equations.

Finally, we need the five–dimensional bulk supersymmetry transformations. For the standard embedding, they have been derived in ref. [30, 36]. To adapt these transformations to our situation, they should be modified in two ways. First, one should apply the replacement (4.9) as discussed. Secondly, one should incorporate the gaugino condensate and the definition (4.11), as in the analogous 11–dimensional

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equations (2.15) and (2.16). This leads to

$$\delta\psi_\alpha^A = \nabla_\alpha\epsilon^A + \frac{\sqrt{2}i}{8}\left(\gamma_\alpha{}^{\beta\gamma} - 4\delta_\alpha^\beta\gamma^\gamma\right) b_{5i}\mathcal{F}^i_{\beta\gamma}\epsilon^A - (P_\alpha{}^A{}_B + S_\alpha{}^A{}_B)\epsilon^B - $$
$$- \frac{\sqrt{2}}{12}V^{-1}b_5^i\alpha_i^{(n)}\gamma_\alpha\,\tau_3{}^A{}_B\epsilon^B \tag{4.14}$$

$$\delta\lambda^{xA} = b_{5i}^x\left(-\frac{1}{2}i\gamma^\alpha\partial_\alpha b_5^i\epsilon^A - \frac{1}{2\sqrt{2}}\gamma^{\alpha\beta}\mathcal{F}^i_{\alpha\beta}\epsilon^A - \frac{i}{2\sqrt{2}}V_5^{-1}\alpha^{(n)i}\tau_3{}^A{}_B\epsilon^B\right) \tag{4.15}$$

$$\delta\zeta^A = -iQ_\alpha{}^A{}_B\gamma^\alpha\epsilon^B - \frac{i}{\sqrt{2}}b_5^i\alpha_i^{(n)}V_5^{-1}\tau_3{}^A{}_B\epsilon^B \tag{4.16}$$

in the interval $z_n \leq |z| \leq z_{n+1}$, where $n = 0, \ldots, N$. Here τ_i with $i = 1, 2, 3$ are the Pauli spin matrices and we have defined

$$P_\alpha{}^A{}_B = \begin{pmatrix} \frac{\sqrt{2}i}{96}V_5\epsilon_{\alpha\beta\gamma\delta\epsilon}G^{\beta\gamma\delta\epsilon} & V_5^{-1/2}\tilde{X}_\alpha \\ -V_5^{-1/2}\bar{\tilde{X}}_\alpha & -\frac{\sqrt{2}i}{96}V_5\epsilon_{\alpha\beta\gamma\delta\epsilon}G^{\beta\gamma\delta\epsilon} \end{pmatrix}^A{}_B$$
$$Q_\alpha{}^A{}_B = \begin{pmatrix} \frac{\sqrt{2}i}{48}V_5\epsilon_{\alpha\beta\gamma\delta\epsilon}G^{\beta\gamma\delta\epsilon} - \frac{1}{2}V_5^{-1}\partial_\alpha V_5 & V_5^{-1/2}\tilde{X}_\alpha \\ V_5^{-1/2}\bar{\tilde{X}}_\alpha & \frac{\sqrt{2}i}{48}V_5\epsilon_{\alpha\beta\gamma\delta\epsilon}G^{\beta\gamma\delta\epsilon} + \frac{1}{2}V_5^{-1}\partial_\alpha V_5 \end{pmatrix}^A{}_B . \tag{4.17}$$

The contribution from the condensate is encoded in $S_\alpha{}^A{}_B$ with the non–zero components

$$(S_{11})^1{}_2 = -(\bar{S}_{11})^2{}_1 = -\frac{3\kappa_5^2}{32\sqrt{2}\pi\alpha_{\rm GUT}}V_5^{-1/2}\left[\Lambda^{(1)}\delta(x^{11}) + \Lambda^{(2)}\delta(x^{11} - \pi\rho)\right] . \tag{4.18}$$

This concludes our review of the five–dimensional effective action. Let us now discuss the vacuum solution of this theory in the absence of a condensate. This solution is a BPS multi–domain wall with $N + 2$ worldvolumes that constitutes the appropriate background for a reduction to four dimensions. It is a straightforward generalization of the solution found in ref. [30, 36] and it can be most easily obtained, in its linearized version, by just rewriting the 11–dimensional solution (2.6)–(2.10) in terms of five–dimensional fields. Moreover, it satisfies the Killing spinor equations $\delta\psi_\alpha^A = 0$, $\delta\lambda^{xA} = 0$ and $\delta\zeta^A = 0$ for a certain Killing spinor ϵ^A as long as the condensate vanishes. Once the condensate is switched on, the only supersymmetry transformation that changes is the one for ψ_{11}^A. As in 11 dimensions, we can try to modify the spinor ϵ^A to $\epsilon^A + \delta\epsilon^A$ so as to compensate for this change. This leads to

$$\partial_{11}(\delta\epsilon^1) = (S_{11})^1{}_2\,\epsilon^2 . \tag{4.19}$$

and the equation for $\delta\epsilon^2$ obtained by conjugation. Again, this equation has a local solution everywhere. The existence of a global solution, however, requires

$$\Lambda^{(1)} + \Lambda^{(2)} = 0 \tag{4.20}$$

which is the five–dimensional analog of eq. (2.23). Hence, we have rediscovered the global breaking mechanism from a five–dimensional viewpoint.

Now, we promote the moduli of the multi–domain wall solution to four–dimensional fields. Then, the sources in the Bianchi identity (4.13) become non–vanishing. The solution for $\tilde{X}_\alpha$ then takes the form

$$\tilde{X}_\alpha = \tilde{X}_\alpha^{(W)} + \tilde{X}_\alpha^{(\chi)}\,, \tag{4.21}$$

where $\tilde{X}_\alpha^{(W)}$ accounts for the sources $J_\mu^{(i)}$ related to the superpotentials on the orbifold planes and $\tilde{X}_\alpha^{(\chi)}$ originates from the condensate sources. The solution for $\tilde{X}_\alpha^{(\chi)}$ is given by

$$\tilde{X}_{11}^{(\chi)} = -\frac{\kappa_5^2}{32\sqrt{2}\pi^2\rho\alpha_{\rm GUT}}\left(\Lambda^{(1)} + \Lambda^{(2)}\right) \tag{4.22}$$

$$\tilde{X}_{\mu}^{(\chi)} = -\frac{\kappa_5^2}{32\sqrt{2}\pi\alpha_{\rm GUT}}\partial_\mu\left(\Lambda^{(1)} - \frac{|x^{11}|}{\pi\rho}(\Lambda^{(1)} + \Lambda^{(2)})\right)\,. \tag{4.23}$$

Then, the gaugino condensate potential arises from inserting $\tilde{X}_{11}^{(\chi)}$ into the perfect square (4.12). We find $W_{\rm gaugino} \sim \Lambda^{(1)} + \Lambda^{(2)}$ as before. Hence, for the "physical" case $\Lambda^{(1)} = 0$ and $\Lambda^{(2)} \neq 0$, we obtain the previous result (3.8) for the superpotential in terms of the gauge kinetic function. Once more we see that the potential is unchanged, at this level, by the three–branes that originate from the five–branes. From the five–dimensional viewpoint, the reason is that these three–branes decouple from the crucial Bianchi identity (4.13) and the equation of motion for $\tilde{X}_\alpha$, as we have stated earlier.

5. The four–dimensional effective action

To proceed further, we need some information about the four–dimensional $\mathcal{N} = 1$ effective action associated with our vacua. In particular, we need to know the gauge kinetic function that enters the gaugino condensate potential (3.8). The main goal of this section is, therefore, to derive the matter field part of the low energy action. To put these results into the correct context we will, however, start by explaining the general structure of the low energy actions derived from our vacua [41].

The field content of the action splits into $N + 2$ sectors coupled to one another only gravitationally. Two of those sectors arise from the E_8 gauge multiplets on the orbifold planes while the other N originate from the degrees of freedom on the five–brane worldvolume theories. All of these sectors have $\mathcal{N} = 1$ supersymmetry. Let us first explain the two "conventional" E_8 sectors. We have considered internal gauge bundles V_i, $i = 1, 2$ with gauge groups G_i in these two sectors. The surviving low energy gauge groups H_i are the commutants of G_i in E_8. In general, from each boundary i, we have gauge matter fields in those representations R of H_i that appear in the decomposition $\mathbf{248}_{E_8} \to \sum_{S,R}(S,R)$ of the adjoint of E_8 under $G_i \times H_i$. The number of families, for a certain representation R, is given by the dimension of $H^1(X, V_{iS})$ where V_{iS} is the vector bundle V_i in the representation S. We stress again that, for the class of vacua we are considering here, we expect many examples where the chiral asymmetry takes the "physical" value three.

From the five–branes, we get additional gauge groups $\mathcal{H}^{(n)}$, $n = 1, \dots N$. Generically, these groups are given by $\mathcal{H}^{(n)} = \mathrm{U}(1)^{g_n}$ where g_n is the genus of the cycle $C^{(n)} \in X$ on which the n–th five–brane wraps. They can, however, enhance to non–Abelian groups, typically unitary groups, when five–branes overlap or a curve $C^{(n)}$ degenerates. In any case, the low energy gauge group is enlarged to $H = H_1 \times H_2 \times \mathcal{H}^{(1)} \times \cdots \times \mathcal{H}^{(N)}$.

Various sets of moduli arise in the low energy theory. First of all, we have the Kähler and complex structure moduli of the Calabi–Yau space along with their superpartners. From the gauge bundles V_i we get two sets of bundle moduli. Furthermore, we have moduli describing the position of the five–branes in the orbifold direction as well as moduli that parameterize the moduli space of the five–brane curve within the Calabi–Yau space.

It is quite possible that reasonable observable sectors could arise from the five–brane worldvolume, although explicit models along these lines have not yet been explored. In this paper, we adopt the more "conservative" viewpoint that one of the E_8 sectors, say the one on the first orbifold plane, constitutes the observable sector while all other sectors are hidden. We have shown [46] that low energy grand unification groups (for example $H_1 = \mathrm{SU}(5)$) with three generations can easily be obtained in this sector due to the freedom introduced by the presence of five–branes. We would then like to study the low energy (matter field) action in this sector and supersymmetry breaking induced by the other E_8 sector. It is likely that the hidden sectors from five–branes can contribute to supersymmetry breaking via gaugino condensation as well under certain circumstances. We postpone the investigation of this question to a future publication [57].

Let us now be more specific about the ingredients we need to discuss the effective theory. For the observable gauge matter, we focus on a specific irreducible representation R of the gauge group H_1. Suppose that R appears as the product $S \times R$ in the decomposition of $\mathbf{248}_{E_8}$ under $G_1 \times H_1$. Then there will be $\dim(H^1(X, V_{1S}))$ families of this type which we denote by $C^{Ip}(R)$. Here $I, J, K, \dots = 1, \dots, \dim(H^1(X, V_{1S}))$ is the family index and $p, q, \dots = 1, \dots, \dim(R)$ the representation index of R. We also need to introduce a basis $\{u_I^x(R)\}$ of $H^1(X, V_{1S})$, where $x, y, \dots = 1, \dots, \dim(S)$ is the representation index of S, and generators $T_{xp}(R)$ corresponding to the (R,S)–part in the decomposition of $\mathbf{248}_{E_8}$. The complex conjugate of these generators is denoted by $T^{xp}(R)$ and the normalization is chosen such that $\mathrm{tr}(T_{xp}(R)T^{yq}(R)) = \delta_x^y \delta_p^q$. As an example, consider a gauge bundle V_1 with gauge group $G_1 = \mathrm{SU}(4)$. Then we have a low energy theory with the grand unification group $H_1 = \mathrm{SO}(10)$. The corresponding decomposition of the adjoint of E_8 under $\mathrm{SU}(4) \times \mathrm{SO}(10)$ reads

$$\mathbf{248} = (\mathbf{15}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{45}) \oplus (\mathbf{6}, \mathbf{10}) \oplus (\mathbf{4}, \mathbf{16}) \oplus (\bar{\mathbf{4}}, \bar{\mathbf{16}})\,. \tag{5.1}$$

The relevant representations are then $(S, R) = (\mathbf{4}, \mathbf{16})$ for the families, $(S, R) = (\bar{\mathbf{4}}, \bar{\mathbf{16}})$ for the anti–families and $(S, R) = (\mathbf{6}, \mathbf{10})$ for the Higgs fields.

Also, we need to introduce explicitly some of the moduli. Most notably, there are the averaged Calabi–Yau volume V and the radius R of the orbifold in units of v and ρ, respectively. The $(1, 1)$–moduli a^i of the Calabi–Yau space, where $i, j, k, \ldots = 1, \ldots, h^{1,1}$, are defined as $\omega = a^i \omega_i$. Here $\omega_{a\bar{b}} = -ig^{(\mathrm{CY})}_{a\bar{b}}$ is the Kähler form. As a function of the $(1, 1)$–moduli, the volume is given by $V = V(a) = \mathcal{K}(a)/6$ with the Kähler potential $\mathcal{K}(a) = d_{ijk} a^i a^j a^k$ and the Calabi–Yau intersection numbers d_{ijk}. We also note that the metric (2.9) can be expressed as

$$G_{ij}(a) = -\frac{1}{2} \partial_i \partial_j \ln \mathcal{K}(a)\,. \tag{5.2}$$

An important role will be played by the moduli $z^n \in [0, 1]$, where $n = 1, \ldots N$, that specify the position of the five–branes in the orbifold direction.

Introducing the conventional four–dimensional chiral fields S, T^i as well as the chiral fields Z^n, we have

$$\mathrm{Re}(S) = V\,, \qquad \mathrm{Re}(T^i) = Ra^i\,, \qquad \mathrm{Re}(Z^n) = z^n\,. \tag{5.3}$$

Another relevant quantity is the metric

$$G^{(R)}_{IJ}(a) = \frac{1}{vV} \int_X \sqrt{g^{(\mathrm{CY})}} g^{(\mathrm{CY})a\bar{b}} u_{Iax}(R) u^x_{J\bar{b}}(R) \tag{5.4}$$

on the moduli space $H^1(X, V_{1S})$. In general, this metric is a function of the Kähler moduli a^i, as we have indicated, as well as a function of the complex structure and bundle moduli. Finally, we need the Yukawa–couplings

$$\lambda^{(R_1 R_2 R_3)}_{IJK} = \int_X \Omega \wedge u^x_I(R_1) \wedge u^y_J(R_2) \wedge u^z_K(R_3) f^{(R_1 R_2 R_3)}_{xyz}\,, \tag{5.5}$$

where $f^{(R_1 R_2 R_3)}_{xyz}$ projects onto the singlet in $R_1 \times R_2 \times R_3$ (if any). As an example, for the above SO(10) case, the relevant products are of course $\mathbf{10} \times \mathbf{16} \times \mathbf{16}$ and $\mathbf{10} \times \bar{\mathbf{16}} \times \bar{\mathbf{16}}$. As is well known, these Yukawa–couplings are quasi–topological, depending on the complex structure moduli and the Dolbeault cohomology classes of the $u^x_I(R)$ only.

For the explicit computation, it is useful to write the metric of our vacuum solution in terms of the moduli introduced above. One finds from eq. (2.6)–(2.8)

$$g_{\mu\nu} = V^{-1} R^{-1} \left(1 + \frac{2\sqrt{2}}{3} b_i a^i \right) g^{(4)}_{\mu\nu} \tag{5.6}$$

$$g_{a\bar{b}} = i \left(a^i - \frac{4\sqrt{2}}{3} b_j a^j a^i + \sqrt{2} b^i \right) \omega_{ia\bar{b}}\,, \tag{5.7}$$

where the b_i are the parameters encoding the deformation of the Calabi–Yau space as defined in eq. (2.10) and $g^{(4)}_{\mu\nu}$ is the four–dimensional Einstein–frame metric. The part of the E_8 gauge field strength that gives rise to the matter fields can be written as

$$F_{\mu\bar{b}} = \sqrt{2\pi\alpha_{\rm GUT}}\sum_R u^x_{I\bar{b}}(R)\,T_{xp}(R)(D_\mu C(R))^{Ip}\,. \tag{5.8}$$

Using these expressions in the 10–dimensional Yang–Mills action

$$S_{\rm YM} = -\frac{1}{8\pi\kappa^2}\left(\frac{\kappa}{4\pi}\right)^{2/3}\sum_{i=1}^{2}\int_{M_{10}^{(i)}}\sqrt{-g}\,{\rm tr}F^{(i)2} \tag{5.9}$$

one can compute the relevant low–energy quantities. In the following, we suppress the representation R as well as the corresponding group indices in order to simplify the notation. Instead, one can think of the indices $I,J,K\ldots$ as running over the various relevant representations as well as over the families. We find for the gauge kinetic function $f^{(1)}$ of the observable group $H^{(1)}$ and the gauge kinetic function $f^{(2)}$ of the hidden group $H^{(2)}$ [41]

$$\begin{aligned} f^{(1)} &= S+\epsilon T^i\left(\beta_i^{(0)}+\sum_{n=1}^{N}(1-Z^n)^2\beta_i^{(n)}\right) && (5.10)\\ f^{(2)} &= S+\epsilon T^i\left(\beta_i^{(N+1)}+\sum_{n=1}^{N}(Z^n)^2\beta_i^{(n)}\right). && (5.11)\end{aligned}$$

The Kähler potential for the observable matter fields has the structure

$$\begin{aligned} K_{\rm matter} &= Z_{IJ}\bar{C}^I C^J && (5.12)\\ Z_{IJ} &= e^{-K_T/3}\left[K_{BIJ}-\frac{\epsilon}{S+\bar{S}}\tilde{\Gamma}^i_{BIJ}\sum_{n=0}^{N+1}(1-z^n)^2\beta_i^{(n)}\right], && (5.13)\end{aligned}$$

with the $(1,1)$ Kähler potential and Kähler metric

$$\begin{aligned} K_T &= -\ln\left(\frac{1}{6}d_{ijk}(T^i+\bar{T}^i)(T^j+\bar{T}^j)(T^k+\bar{T}^k)\right) && (5.14)\\ K_{Tij} &= \frac{\partial^2 K_T}{\partial T^i\partial\bar{T}^j} && (5.15)\end{aligned}$$

and the following quantities associated with the vector bundle

$$\begin{aligned} K_{BIJ} &= G_{IJ}(T+\bar{T}) && (5.16)\\ \Gamma^i_{BIJ} &= K_T^{ij}\frac{\partial K_{BIJ}}{\partial T^j} && (5.17)\\ \tilde{\Gamma}^i_{BIJ} &= \Gamma^i_{BIJ}-(T^i+\bar{T}^i)Z_{BIJ}-\frac{2}{3}(T^i+\bar{T}^i)(T^k+\bar{T}^k)K_{Tkj}\Gamma^j_{BIJ}\,. && (5.18)\end{aligned}$$

Note that the metric K_{BIJ} equals the bundle metric (5.4) viewed as a function of $T^i + \bar{T}^i$. Finally, for the superpotential of the observable matter, we have the usual expression

$$W_{\rm matter} = \frac{1}{3}\tilde{Y}_{IJK} C^I C^J C^K \,, \tag{5.19}$$

where

$$\tilde{Y}_{IJK} = 2\sqrt{2\pi\alpha_{\rm GUT}}\,\lambda_{IJK} \,. \tag{5.20}$$

We remark that the Kähler potential for hidden matter can easily be obtained from eq. (5.13) by the replacement $z^n \to 1 - z^n$.

We can now insert the result (5.11) for the hidden sector gauge kinetic function into eq. (3.8). This leads to the explicit gaugino condensate potential

$$W_{\rm gaugino} = h \exp\left[-\frac{6\pi}{b\alpha_{\rm GUT}}\left(S + \epsilon T^i \sum_{n=0}^{N+1} (Z^n)^2 \beta_i^{(n)}\right)\right] \tag{5.21}$$

for a single condensate. Although the presence of five–branes did thus far not effect the condensate potential, we see that this final step introduces a five–brane dependent modification resulting from the threshold correction to the gauge kinetic function. As a result, the potential (5.21) not only depends on the dilaton S and the T–moduli, but also on the five–brane position moduli Z^n.

6. Phenomenological issues

In this section we discuss some phenomenological issues related to supersymmetry breaking and the effective four–dimensional matter field action that we have computed. Our emphasis will be on the consequences for soft supersymmetry breaking terms.

We will not attempt to construct explicit gaugino condensate models based on the superpotential (5.21), since that would lead us into complicated model building. Instead, we will parameterize supersymmetry breaking in terms of the auxiliary fields of the moduli. Usually, this is done using the auxiliary fields associated to the dilaton and the T–moduli [60, 61]. The presence of the five–brane position moduli Z^n in the gaugino condensate superpotential, however, suggests that one should, in general, also allow their auxiliary components to contribute to supersymmetry breaking. Hence, we will analyze supersymmetry breaking that can be parameterized by auxiliary fields (F^S, F^i, F^n) corresponding to the moduli (S, T^i, Z^n), respectively.

The starting point is the Kähler potential

$$K = \kappa_P^{-2} K_{\rm mod} + Z_{IJ}\bar{C}^I C^J \,, \tag{6.1}$$

where $K_{\rm mod}$ is the Kähler potential of the moduli depending on S, T^i, Z^n and the other moduli and Z_{IJ} was given in eq. (5.13). The four–dimensional Newton constant κ_P is defined in terms of 11–dimensional quantities as $\kappa_P^2 = \kappa^2/2\pi\rho v$. One also needs the gauge kinetic function $f^{(1)}$, eq. (5.10) and the Yukawa couplings $\tilde{Y}_{IJK}$ in the superpotential (5.20). After integrating out the moduli, the effective Yukawa coupling will be

$$Y_{IJK} = e^{K_{\rm mod}/2}\tilde{Y}_{IJK}\,. \tag{6.2}$$

It will be useful for our discussion to explicitly list the general expressions for the soft masses [58, 59, 60]. One has

$$m_{3/2}^2 = \frac{1}{3}K_{{\rm mod},a\bar{b}}F^a\bar{F}^{\bar{b}} \tag{6.3}$$
$$m_{1/2} = \frac{1}{2{\rm Re}(f^{(1)})}F^a\partial_a f^{(1)} \tag{6.4}$$
$$m_{I\bar{J}}^2 = m_{3/2}^2 Z_{I\bar{J}} - F^a\bar{F}^{\bar{b}}R_{a\bar{b}I\bar{J}} \tag{6.5}$$
$$A_{IJK} = F^a\left(\partial_a Y_{IJK} + \frac{1}{2}K_{{\rm mod},a}Y_{IJK} - \Gamma^N_{a(I}Y_{JK)N}\right), \tag{6.6}$$

where

$$R_{a\bar{b}I\bar{J}} = \partial_a\bar{\partial}_{\bar{b}}Z_{I\bar{J}} - \Gamma^N_{aI}Z_{N\bar{L}}\bar{\Gamma}^{\bar{L}}_{\bar{b}\bar{J}} \tag{6.7}$$
$$\Gamma^N_{aI} = Z^{N\bar{J}}\partial_a Z_{\bar{J}I}\,. \tag{6.8}$$

Here $m_{3/2}$ is the gravitino mass, $m_{1/2}$ is the gaugino mass, $m^2_{I\bar{J}}$ are the scalar soft masses and A_{IJK} are the trilinear soft couplings. Indices $a, b, c, \ldots$ run over all moduli fields.

There are two aspects of our scenario that make an analysis of soft terms interesting; that is, the possibility of supersymmetry breaking in the direction of the five–brane position moduli and the presence of the order ϵ correction to the effective action. Both aspects might influence the magnitude of the soft terms compared to perturbative heterotic string, as we will discuss later on in more detail. For standard–embedding vacua of heterotic M–theory and a simple model with one family, soft terms in the presence of ϵ corrections have been already studied [15, 21, 22].

Before we come to that, we would like to address the problem of universality of soft breaking parameters. In fact, universality might provide a stronger phenomenological constraint than just the overall size of soft terms. We remark that for a perfectly universal model (at the unification scale) we should have $m^2_{I\bar{J}} \sim Z_{I\bar{J}}$ and $A_{IJK} \sim Y_{IJK}$. The obstruction to universality results from the last term in the expressions for $m^2_{I\bar{J}}$ and A_{IJK} in eq. (6.5) and (6.6). Of course, there is nothing wrong with choosing the moduli so as to make these last terms universal as well (something one expects to be able to do for a sufficiently large number of moduli).

However, given the accuracy at which universality holds, this requires a significant fine tuning in moduli space. Therefore, it is much more desirable to have a reason for universality to occur in a given model. The usual assumption along these lines is that supersymmetry is broken in the direction of moduli that are flavor–blind. To discuss this possibility, let us neglect the ϵ corrections to our effective action for the moment and concentrate on the lowest order part. Clearly, the lowest order part of the matter field Kähler potential is independent on S and Z^n. Therefore, if supersymmetry breaking occurs in the dilaton direction the soft terms will be universal to that order [60]. The same is true if some of the auxiliary fields F^n associated to the five–brane positions are non–vanishing. This last option represents a new breaking pattern that is directly related to the presence of five–branes in our vacua. What is the reason for this structure from an 11–dimensional viewpoint? Both, the T–moduli and the matter fields arise as zero modes associated to the shape of the Calabi–Yau space. Hence supersymmetry breaking in the T^i directions is generally not flavor–blind. On the other hand, the moduli Z^n arise as zero–modes of the five–brane worldvolume theories and, hence, do not distinguish between the generations to zeroth order.

Let us now discuss the effect of the ϵ corrections, which we have neglected so far. Of course, if ϵ is very small these terms need not concern us much. However, it is well known that ϵ is of order one at the "physical" point where coupling unification can be realized [5]. Hence, those terms are quite possibly subject to the universality constraint as well. Unfortunately, from eq. (5.13) we see that the ϵ corrections to the Kähler metric depend on S as well as on Z^n. In addition, the zero and first order contributions to Z_{IJ} are generically not proportional to each other. This implies that, unlike for the lowest order case, there is no specific pattern of supersymmetry breaking that will lead to universality, in general. Note, that also the F^S scenario will not work in this case. Let us again discuss this from an 11–dimensional point of view. To zeroth order, moduli were flavor–sensitive or flavor–blind depending on whether or not they arise as zero modes of the Calabi–Yau space. This does not remain true once the ϵ corrections are taken into account. These corrections correspond to an orbifold dependence of the 11–dimensional background that arises from the sources on the orbifold planes and the five–branes. This dependence encodes "interaction" across the orbifold and leads, for example, to a coupling of the five–brane moduli to the matter fields. This coupling appears despite the fact that the matter fields and the five–brane moduli are separated in the orbifold direction. We remark that one expects such corrections quite generally in vacua that contain branes although they need not to be necessarily large.

One way out of the universality problem is to find a way to make the zeroth and first order part of Z_{IJ} proportional. There is, indeed, a simple way to do that, namely to choose a Calabi–Yau space with one Kähler modulus only ($h^{1,1} = 1$). Such Calabi–Yau spaces exist, the quintic hypersurface in CP^4 being the simplest example. Moreover, as our previous experience shows [46], with the freedom introduced by allowing non–standard embedding and five–branes the construction of three–generation models on such Calabi–Yau spaces might be considerably easier than for the restrictive standard–embedding. Let us, therefore, analyze this case in more detail. Since we have only one T–modulus, the metric K_{BIJ} in eq. (5.17) can be written as

$$K_{BIJ} = \frac{3}{(T+\bar{T})^2} H_{IJ} \tag{6.9}$$

with some T–independent metric H_{IJ}. We then find for the matter field Kähler metric

$$Z_{IJ} = \left[\frac{3}{T+\bar{T}} + \frac{\epsilon\zeta}{S+\bar{S}} \right] H_{IJ}\,, \tag{6.10}$$

where

$$\zeta = \beta^{(0)} + \sum_{n=1}^{N} (1-z^n)^2 \beta^{(n)}\,. \tag{6.11}$$

The relevant part of the moduli Kähler potential then reads

$$K_{\rm mod} = -\ln(S+\bar{S}) - 3\ln(T+\bar{T}) + K_5\,, \tag{6.12}$$

where K_5 is the Kähler potential for the five–brane moduli Z^n. Now S, T as well as the five–brane moduli Z^n enter the matter Kähler potential in a generation–independent way and we expect universal soft masses as long as supersymmetry is broken in the sector (F^S, F^T, F^n) (which was our original assumption). More specifically we get

$$m_{3/2}^2 = \frac{|F^S|^2}{3(S+\bar{S})^2} + \frac{|F^T|^2}{(T+\bar{T})^2} + \frac{1}{3} K_{5n\bar{m}} F^n \bar{F}^{\bar{m}} \tag{6.13}$$

for the gravitino mass. The gaugino mass is given by

$$m_{1/2} = m_{1/2}^{(0)} + \epsilon m_{1/2}^{(1)}\,, \tag{6.14}$$

with

$$m_{1/2}^{(0)} = \frac{F^S}{2(S+\bar{S})} \tag{6.15}$$

$$m_{1/2}^{(1)} = \frac{\zeta F^T}{2(S+\bar{S})} - \frac{(T+\bar{T})\zeta F^S}{2(S+\bar{S})^2} + \frac{T\zeta_n F^n}{2(S+\bar{S})}\,. \tag{6.16}$$

For the scalar soft masses we get

$$m_{IJ}^2 = (m^{(0)^2} + \epsilon m^{(1)^2}) H_{IJ}\,, \tag{6.17}$$

with

$$m^{(0)\,2} = \frac{|F^S|^2}{(S+\bar{S})^2(T+\bar{T})} + \frac{K_{5n\bar{m}}F^n\bar{F}^{\bar{m}}}{T+\bar{T}} \tag{6.18}$$

$$\begin{aligned} m^{(1)\,2} &= -\frac{5\zeta|F^S|^2}{3(S+\bar{S})^3} + \frac{2\zeta\mathrm{Re}(F^S\bar{F}^{\bar{T}})}{(S+\bar{S})^2(T+\bar{T})} + \frac{\zeta K_{5n\bar{m}}F^n\bar{F}^{\bar{m}}}{3(S+\bar{S})} - \\ &\quad -\frac{\zeta_{n\bar{m}}F^n\bar{F}^{\bar{m}}}{S+\bar{S}} + \frac{2\mathrm{Re}(F^S\bar{F}^{\bar{n}}\zeta_{\bar{n}})}{(S+\bar{S})^2} - \frac{2\mathrm{Re}(F^T\bar{F}^{\bar{n}}\zeta_{\bar{n}})}{(T+\bar{T})(S+\bar{S})}\,. \end{aligned} \tag{6.19}$$

Finally, for the trilinear soft terms we find

$$A_{IJK} = (A^{(0)} + \epsilon A^{(1)})Y_{IJK}\,, \tag{6.20}$$

where

$$A^{(0)} = -\frac{F^S}{S+\bar{S}} + F^n K_{5n} \tag{6.21}$$

$$A^{(1)} = -\frac{\zeta F^T}{S+\bar{S}} + \frac{(T+\bar{T})\zeta F^S}{(S+\bar{S})^2} - \frac{(T+\bar{T})\zeta_n F^n}{S+\bar{S}}\,. \tag{6.22}$$

In these equations, ζ contains the five–brane moduli Z^n and is given by eq. (6.11). We also used an index notation for derivatives with respect to Z^n, for example $\zeta_n = \partial\zeta/\partial Z^n$. We see that, by means of a simple topological assumption and supersymmetry breaking in the (F^S, F^T, F^n) sector, we have achieved universality to linear order in ϵ (that is, to the order we can calculate). Depending on the specific breaking pattern, the size of the various soft terms is either of order $m_{3/2}$ or of order $\epsilon m_{3/2}$. Given that ϵ still has to be somewhat smaller than one in order to stay within the limits of our approximations, the second option implies a certain suppression. Let us discuss some examples. Consider supersymmetry breaking in the F^T direction. Then the gaugino mass is of order $\epsilon m_{3/2}$, an observation that was first made in ref. [15]. Also, the trilinear soft terms are of that order while the scalar soft masses still vanish [21, 22]. As an alternative possibility, consider supersymmetry breaking in the directions F^n of the five–brane moduli. The gaugino masses then remain at the order $\epsilon m_{3/2}$ while trilinear soft terms and scalar masses are now of order $m_{3/2}$. In general, we see that the structure of our models provides a direct correlation between the magnitude of soft masses and the pattern of supersymmetry breaking.

We would like to conclude with some outlook on supersymmetry breaking via gaugino condensation in the type of models considered in this paper. Here, we have only analyzed the simplest case of gaugino condensation in the E_8 sectors. As we have seen, one expects additional hidden sectors in the effective theory that originate from the degrees of freedom on the five–brane worldvolumes. In particular, under certain circumstances [46], these hidden sectors have non–Abelian gauge groups.

Hence, gaugino condensation might also occur in these sectors. Due to the origin of these new gauge groups, their gauge kinetic functions might be quite different from the "conventional" ones in the E_8 sectors. This might shed new light on the generic problems of gaugino condensation [62], such as the stabilization of the dilaton at large field values. We will address these issues in a future publication [57].

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